

Hetton Lyons

Calculation Policy

2022 - 2023

**Policy Overview**

**Principles of the Policy**

One of the key learning principles behind this policy is the concrete pictorial abstract approach (CPA). The concrete-pictorial-abstract approach, is based on research by psychologist Jerome Bruner, and suggests that there are three steps (or representations) necessary for children to develop understanding of a concept.

For children to have a deep understanding of the mathematical concepts being developed, they need to ‘master’ all three phases of the CPA approach. If a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial as an additional scaffold. If a child is working at the abstract stage, ‘proving’ something or ‘working out’ then concrete or pictorial representations could be used to develop a greater depth as pupils articulate their thinking /reasoning. Reinforcement is achieved by going back and forth between these representations. Linking abstract notation to pictorial/concrete representations and then the concrete/pictorial models to an abstract notation.

**Concrete Representation**

The ***enactive stage***. Children are first introduced to an idea/skill/concept by acting it out with real objects, this could include large scale with the pupils themselves and also utilise resourced available in the outdoor environment. This is a ***'hands on'*** stage using real objects linked to real-life and the wider curriculum and/or mathematical equipment, (i.e. counters, cubes, bead string, five and ten frames, Dienes, place value counters etc.) and it is the foundation for ***conceptual understanding.***

**Pictorial Representation**

The ***iconic stage***. A child has sufficiently understood the hands-on, ***concrete*** experiences performed and can now relate them to ***pictorial*** representations, such as a ***diagram*** or ***pictures*** of the problem. ***Pictorial*** representations, such as the bar model, can also be used to ***scaffold*** understanding.

**Abstract Representation**

The ***symbolic stage***. A child is now capable of representing problems by using ***abstract*** mathematical notation, for example: 12 ÷ 2 = 6. This is the ultimate mode.

Teachers should understand how each stage of the CPA approach can be used effectively to ***model*** concepts, ***scaffold*** learning and ***record*** thinking:

***Modelling***: teachers to make clear links are made between ***concrete*** representations (which can also be represented pictorially), ***pictorial*** representations (diagrams and pictures, including bar modelling) and ***abstract*** notations.

***Scaffolds***: children to be offered the opportunity to use appropriate ***concrete*** and ***pictorial*** representations to further ***scaffold*** their understanding. The ***scaffolds*** offered, must be familiar and understood by children. Children should be encouraged to consider whether ***scaffolds*** are required and for how long they require them for. ***Concrete*** and ***pictorial*** representations are also supportive when developing children’s depth of knowledge through problem solving and reasoning experiences.

***Recording:***

***Concrete recording***: when children are unable to ***record*** their thinking using ***pictorial*** representation or ***abstract*** notation then their learning can be evidenced through photographic evidence and post-it notes, which detail key information regarding children’s strengths and areas of developments/gaps/misconceptions.

***Pictorial recording***: children to be encouraged to represent their thinking using ***pictorial*** representations, if they are unable to record using ***abstract*** notation. ***Pictorial*** representation also includes the use of the bar model.

***Abstract recording***: this is the ultimate mode but should not be rushed at the expense of true ***conceptual understanding***. Scaffolds, such as missing box calculations, can support children’s transition towards ***abstract***.

**Big Ideas**

The next section of the policy highlights the ‘big ideas’ for each year group and the number facts that children should be able to recall.

**Big Ideas -** This highlights the big ideas in place value, addition and subtraction and multiplication and division for each year group. These big ideas should be a focus throughout the year.

**Derive & recall, mental skills, strategies & methods –** This highlights the mental skills and strategies that children need in each year group. Children should have a ‘Daily Maths’ session each day. The session should last for around 10 -15 minutes and can be part of the Maths lesson or at a separate part in the day. The objectives under the heading of ***derive & recall, mental skills, strategies & methods***should be focused on in these sessions.

**Year 1**

**Year 1 Progression Place Value and Counting**

**Place Value Big Ideas**

* The ***position*** a digit is placed in a number determines its ***value***.
* The ***language*** used to ***name numbers*** does not always expose the ***place value***, for example the word ‘twelve’ does not make it transparent that the value of this number is ten and two. It is important that children develop ***secure understanding of the value of each digit***.
* ***Place value*** is based on ***unitising***: treating a group of things as ***one ‘unit’***. In mathematics, units can be any size, for example units of 1, 2, 5 and 10 are used in money. ***In place value units of 1 (one), 10(ten) and 100 (hundred) are used.***

**Place Value:derive & recall, mental skills, strategies & methods**

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| ***Year 1 Autumn Term:***  ***Place Value and Counting*** | Count ***to 10*** ***then 20*** forwards and backwards, beginning with 0 or 1, or any given number. | Recognise and say numbers in the environment and with personal significance as appropriate  *(i.e. age, door number etc.).* | Recognise, say and identify numerals ***to 10*** then ***20***. | Say the numbers ***0-10 then 0 - 20*** in a random order. | Order numbers ***to 10 then 20*** positioning them on a number track. | Count reliability objects ***up to 20.*** | Estimate and check by counting. | Instantly recognise, without counting (subitise), organised (***up to 10***) and random arrangements (***up to 5***) of small numbers of objects. | Use the language of: equal to, more than, less than (fewer), most, least ***(numbers within 20).*** |

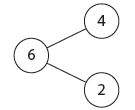
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| * ***Year 1 Spring Term:*** * ***Place Value and Counting*** | Count ***to 40***, forwards and backwards, beginning with 0 or 1, or any given number. | Recognise and say numbers in the environment as appropriate and with personal significance as appropriate *(i.e. age, door number etc.).* | Recognise, say and identify numerals ***to 40.*** | Say the numbers ***0-40*** in a random order. | Count reliability objects ***up to 40.*** | Estimate and check by counting. | Use the language of: equal to, more than, less than (fewer), most, least ***(numbers within 40).*** |

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| ***Year 1 Summer Term:***  ***Place Value and Counting*** | Count ***to and across 100***, forwards and backwards, beginning with 0 or 1, or any given number. | Recognise and say numbers in the environment as appropriate and with personal significance as appropriate *(i.e. age, door number etc.).* | Recognise, say and identify numerals to 100. | Say the numbers ***0-100*** in a random order. | Count reliability objects ***up to 100.*** | Estimate and check by counting. | Use the language of: equal to, more than, less than (fewer), most, least ***(numbers within 100).*** |

**Year 1 Progression: Addition & Subtraction**

**Addition & Subtraction Big Ideas**

* Relating numbers to ***5*** and ***10*** helps develop knowledge of the ***number bonds within 20.*** For example, given 8 + 7, thinking of 7 as 2 + 5 and adding the 2 to 8 to make 10 and then the 5 to total 15.
* Thinking of ***part whole relationships*** is helpful in linking ***addition and subtraction***. For example, where the whole is 6, and 4 and 2 are parts. This means that 4 and 2 together form the whole, which is 6 and 6 subtract 4 leaves the 2 and 6 subtract 2 leaves the 4.



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| * ***Year 1 Autumn Term: Addition & Subtraction: derive & recall, mental skills, strategies & methods*** | * Number pairs with a ***total of 10***,   *(e.g. 3 + 7, what to add to a single-digit number to make 10, e.g. 3 + □ = 10).* | * Addition doubles for all numbers ***to 5 (total of 10)*** then ***10*** (***total of 20***), * *(e.g. 8 + 8).* | * Addition and subtraction facts for all numbers to a ***total of 10***, including ***zero*** * *(e.g. 3 + 4, 8 – 5).* | Identify one more and one less ***within 10 then 20***. | * Add or subtract a pair of ***single-digit numbers***, including ***crossing the tens boundary***, ***within 20***, *(e.g. 4 + 5, 5 + 6, 8 – 3, 9 - 4).*   ***Aggregation****: counting each group and then counting all*  ***Augmentation:*** *counting on from a number, including counting on from the largest number (reorder when adding)*  *partition: bridge through 10 when adding,*  *(e.g. 7 + 5 = 7 + 3 + 2)*  *count on or back in 1s or 2s*  *use knowledge of pairs making 10* | * Add and subtract groups of small ***single-digit numbers***, * *(e.g. 5 – 3 + 2).* * *reorder numbers when adding* * *identify pairs totalling 10 or make doubles* | * Add or subtract a ***single-digit number*** to or from a ***teens number*** , including ***crossing the tens boundary*** ***for subtraction***, ***within 20***, * (e.g. 13 + 5, 12 – 4, 17 – 3).   *reorder numbers when adding, (e.g. put the larger number first*  *partition and combine tens and ones*  *count on or back in 1s, 2s or 10s*   * *partition: bridge through 10 when subtracting, (e.g. 13 - 5 = 13 – 3 – 2)* |

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| * ***Year 1 Spring Term: Addition & Subtraction: derive & recall, mental skills, strategies & methods*** | * Addition doubles for all numbers ***to 5 (total of 10)*** then ***10*** (***total of 20***), * *(e.g. 8 + 8).* | * Addition and subtraction facts for all numbers to a ***total of 20***, including ***zero***,   *(e.g. 13 + 4, 18 – 5).* | * Identify one more and one less, ***within 40***. | * Add a ***multiple of 10*** to a ***single-digit number***, ***within 40***, * *(e.g. 7 + 30).*   *reorder numbers when adding, (e.g. put the larger number first)* | * Add near doubles to with numbers ***to 10***, * *(e.g. 6 + 7).*   *partition: double and adjust, (e.g. 5 + 6 = 5 + 5 + 1)* | * Add and subtract a ***two-digit number*** and ***ones***, including ***crossing the tens boundary***, ***within 40***, * *(e.g. 24 + 3, 24 + 7, 36 – 3, 36 – 8).* | * Add and subtract a ***two-digit number*** and ***tens***, ***within 40***, * *(e.g. 24 + 10, 14 + 20, 36 – 10, 36 – 20).* | Add and subtract two, ***two-digit numbers***, including ***crossing the tens boundary***, ***within 40***,   * *(e.g. 24 + 13, 24 + 17, 36 – 13, 36 –1 8).* |

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| * ***Year 1 Summer Term: Addition & Subtraction: derive & recall, mental skills, strategies & methods*** | * Addition doubles for all numbers ***to 5 (total of 10)*** then ***10*** (***total of 20***), * *(e.g. 8 + 8).* | * Identify one more and one less, ***within 100***. | * Add and subtract ***one-digit*** and ***two-digit numbers*** ***to 20***, including zero, *(e.g. 13 + 4, 18 – 5).* |

**Year 1 Progression Multiplication and Division**

**Multiplication and Division Big Ideas**

* Counting in ***steps of equal sizes*** is based on the big idea of ***‘unitising’***; treating a group of, say, five objects as one unit of five.
* Working with ***arrays*** helps pupils to become aware of the ***commutative property of multiplication***, that 2 × 5 is equivalent to 5 × 2.

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| ***Year 1 Autumn Term: Multiplication & Division: rapid recall facts, mental skills, strategies & methods*** | Doubles of all numbers ***to 10*** and ***corresponding halves***,  *(e.g. double 6).* | Odd and even numbers ***to 20,*** *(e.g. use patterns of last digits and make links to counting in twos for even numbers,*  *(e.g. 0, 2, 4, 6, 8).* | Count on from and back to zero in ***twos,*** *(e.g. use patterns of last digits and make links to even numbers, (e.g. 0, 2, 4, 6, 8).* | Count on from and back to zero in ***fives,*** *(e.g. use patterns of last digits, (e.g. 0 and 5).* | Count on from and back to zero in ***tens,*** *(e.g. use patterns of last digits, (e.g. 0).* | Solve one-step problems involving multiplication and division, by calculating the answer using ***concrete objects, pictorial representations*** and ***arrays*** with the support of the teacher. |

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| ***Year 1 Spring Term: Multiplication & Division: rapid recall facts, mental skills, strategies & methods*** | Doubles of all numbers ***to 10*** and ***corresponding halves***,  *(e.g. double 6).* | Odd and even numbers ***to 40,*** *(e.g. use patterns of last digits and make links to counting in twos for even numbers,*  *(e.g. 0, 2, 4, 6, 8).* | Count in multiples of ***twos,*** *(e.g. use patterns of last digits and make links to even numbers, (e.g. 0, 2, 4, 6, 8).* | Count in multiples of ***fives,*** *(e.g. use patterns of last digits, (e.g. 0 and 5).* | Count in multiples of ***tens,*** *(e.g. use patterns of last digits, (e.g. 0).* | solve one-step problems involving multiplication and division, by calculating the answer using ***concrete objects, pictorial representations*** and ***arrays*** with the support of the teacher |

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| ***Year 1 Summer Term: Multiplication & Division: rapid recall facts, mental skills, strategies & methods*** | Doubles of all numbers ***to 10*** and ***corresponding halves***,  *(e.g. double 6).* | Odd and even numbers ***to 100,*** *(e.g. use patterns of last digits and make links to counting in twos for even numbers,(e.g. 0, 2, 4, 6, 8).* | Count in multiples of ***twos,*** *(e.g. use patterns of last digits and make links to even numbers, (e.g. 0, 2, 4, 6, 8).* | Count in multiples of ***fives***, *(e.g. use patterns of last digits, (e.g. 0 and 5).* | Count in multiples of ***tens,*** *(e.g. use patterns of last digits, (e.g. 0).* | Solve one-step problems involving multiplication and division, by calculating the answer using ***concrete objects, pictorial representations*** and ***arrays*** with the support of the teacher. |

**Year 2**

**Year 2 Progression Place Value and Counting**

**Place Value Big Ideas**

* The ***position (place)*** of a ***digit*** in a number ***determines its value***. Hence the term ***place value.***

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| Count in steps of ***2*** from 0, forwards and backwards, ***within 100***. | Count in steps of ***5*** from 0, forwards and backwards, ***within 100***. | Count in steps of ***10*** from 0, forwards and backwards from any number, ***within 100.*** | Count in steps of ***3*** from 0, forwards and backwards. | Compare and order numbers from ***0 to 100.*** | Demonstrate an understanding of place value, with apparatus to support still if necessary, ***within 100***,  *(e.g. stating the difference between the tens and ones and writing statements using > & < symbols)* | Recognise the place value of each digit in a ***two digit number, within 100***. | Partition ***two-digit numbers*** into *different* combinations of tens and ones, this may include apparatus, ***within 100***,  *(e.g. 23 is the same as 2 tens and 3 ones, same as 1 ten and 13 ones)* |

**Year 2 Progression Addition & Subtraction**

**Addition & Subtraction Big Ideas**

* Understanding that ***addition*** of two or more numbers can be done in ***any order*** is important to support children’s fluency. When adding two numbers it can be ***more efficient*** to put the ***larger number first***. For example, given 3 + 8 it is easier to calculate 8 + 3.
* When adding ***three or more numbers*** it is helpful to ***look for pairs of numbers*** that are easy to add. For example, given 5 + 8 + 2 it is easier to add 8 + 2 first than to begin with 5 + 8.
* Understanding the importance of the ***equals sign*** meaning ***‘equivalent to’*** (i.e. that 6 + 4 = 10, 10 = 6 + 4 and 5 + 5 = 6 + 4 are all valid uses of the equals sign) is crucial for later work in algebra. ***Empty box problems*** can support the development of this key idea. Correct use of the equals sign should be reinforced at all times. ***Altering where the equals sign is placed*** develops ***fluency and flexibility***.

**Addition & Subtraction: derive & recall, mental skills, strategies & methods**

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| * Addition and subtraction facts for all numbers ***to 20***,   *(e.g. 9 + 8, 17 – 9, drawing on knowledge of inverse operations)* | Derive and use related facts ***to 100***,  *(e.g. 3 + 4 = 7, 30 + 40 = 70 etc.)* | All pairs of ***multiples of 10*** (to a ***total of 100***),  *(e.g. 30 + 70, or 60 + □ = 100)* | What must be added to any ***two-digit number*** to make the ***next multiple of 10***,  *(e.g. 52 + □ = 60)* | * Addition doubles for all numbers ***to 20 (total of 40)***   *(e.g. 18 +1 8).* | Addition doubles for all ***multiples of 10 to 100***,  *(e.g. 40 + 40)* | Add near doubles, ***within 100***,  *(e.g. 13 + 14, 39 + 40)* | Add & subtract any ***single-digit number*** to or from a ***multiple of 10***, ***within 100***,  *(e.g. 60 + 5, 80 - 5)* |

**Written Procedures**

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| Add or subtract a ***single-digit numbe***r to or from a ***two-digit number***, within 100, ***without crossing the tens boundary***, (e.g. 23 + 5, 57 – 3) | Add or subtract a ***single-digit number*** to or from a ***two-digit number***, ***within 100***, ***crossing the tens boundary***,  *(e.g. 28 + 5, 52 – 7)* | Add or subtract a ***multiple of 10*** to or from any ***two-digit number***, ***within 100***, *(e.g. 27 + 60, 72 – 50)* | Add or subtract a ***two-digit number*** to or from a ***multiple of 10***, ***within 100***, *(e.g. 50 + 38, 90 – 27)* | Add and subtract ***two-digit numbers*** , ***without crossing the tens boundary***, ***within 100***, *(e.g. 34 + 65, 68 – 35)* | Add and subtract ***two-digit numbers***, ***crossing the tens boundary***, ***within 100***,  *(e.g. 38 + 65, 64 – 35)* | Add 9, 19, 29, 11, 21, 31 etc. |

**Year 2 Progression Multiplication & Division**

**Multiplication & Division Big Ideas**

* It is important that pupils both ***commit multiplication facts to memory*** and also develop an understanding of ***conceptual relationships***. This will aid them in ***using known facts*** to ***work out unknown facts*** and in solving problems.
* Pupils should look for and recognise ***patterns*** within tables and ***connections*** between them (e.g. 5× is half of 10×).
* Pupils should recognise multiplication and division as ***inverse operations*** and use this knowledge to solve problems. They should also recognise division as both ***grouping and sharing***.
* The recognition of ***pattern*** in multiplication helps pupils ***commit facts to memory***, for example doubling twice is the same as multiplying by four, or halving a multiple of ten gives you the related multiple of five.

**Multiplication & Division: rapid recall facts, mental skills, strategies & methods**

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| Doubles of all numbers ***to 20***, and the corresponding even ***halves***,  *(e.g. double 13, halve 26).* | Find half of even numbers ***to 100.***  *use knowledge that halving is the inverse of doubling*  *partition: halve the tens and ones separately, then recombine* | Doubles of ***multiples of 10 to 100***, and the corresponding ***halves.***  *(e.g. double 40, halve 80)*  *use knowledge that halving is the inverse of doubling* | Multiplication facts for the ***2 times-tables***, and the corresponding ***division facts.*** | Multiplication facts for the ***5 times-tables***, and the corresponding ***division facts.*** | Multiplication facts for the ***10 times-tables***, and the corresponding ***division facts.*** | Odd and even numbers ***to 100.*** |

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| Find the total number of objects when they are organised into groups of ***2, 5 or 10.***  *use knowledge of multiplication facts from the 2, 5 and 10 times-tables,*  *(e.g. recognise that there are 15 objects altogether because there are three groups of five)* | Calculate mathematical statements for multiplication and division with the ***2, 3, 5, & 10*** times tables, *(e.g. using arrays).*  *show that multiplication of two numbers can be done in order (commutative) and division of one number by another cannot* | Understand multiplication as repeated addition and arrays.  *show that multiplication of two numbers can be done in order (commutative) and division of one number by another cannot* | Understand division as sharing and grouping. | Know that a division calculation can have a remainder. |

**Year 3**

**Year 3 Progression Place Value and Counting**

**Place Value Big Ideas**

* The ***value*** of a ***digit*** is determined by its ***position in a number***.
* Place value is based on ***unitising***, treating a group of things as ***one ‘unit’***. This generalises to 3 units + 2 units = 5 units (where the units are the same size).

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| Count from 0 in multiples of ***4.*** | Count from 0 in multiples of ***8.*** | Count from 0 in multiples of ***50.*** | Count from 0 in multiples of ***100.*** | Compare and order numbers ***to 1000.*** | Recognise the place value of each digit in a ***three-digit number*** ,  *(e.g. hundreds, tens and ones.)* | Read Roman numerals to 12. | Find 10 more and 10 less than a given number, ***up to 1000.*** | Find 100 more and 100 less than a given number, ***up to 1000.*** |

**Year 3 Progression: Addition & Subtraction**

**Addition & Subtraction Big Ideas**

* Relating numbers to ***5*** and ***10*** helps develop knowledge of the ***number bonds within 20***. For example, given 8 + 7, thinking of 7 as 2 + 5, and adding the 2 and 8 to make 10, then the 5 to 15. This should then be applied when calculating with larger numbers.
* ***Subtraction bonds*** can be thought of in terms of ***addition***: for example, in answering 15 – 8, thinking what needs to be added to 8 to make 15. ***Counting on for subtraction*** is a useful strategy that can also be applied to ***larger numbers***.

**Addition & Subtraction: derive & recall, mental skills, strategies & methods**

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| * Sums and differences of ***multiples of 10***, ***within 100***   *(e.g. 50 + 40, 90 – 30)* | Sums and differences of ***pairs of multiples of 10***, & ***100***, ***including crossing the***  ***tens boundary***, **within 1000**  *(e.g. 50 + 80, 120 – 90, 300 + 400)* | Pairs of ***two-digit numbers*** with a ***total of 100***,  *(e.g. 32 + 68, or 32 + □ = 100)* | Addition doubles of numbers ***1 to 100***, *(e.g. 38 + 38)* | Add and subtract a **near** ***multiple of 10***, including ***crossing the tens boundary***, *(e.g. 56 + 29, 86 – 38)*  *partition: add or subtract a multiple of 10 and adjust,*  *(e.g. 56 + 29 = 56 + 30 – 1, or 86 – 38 = 86 – 40 + 2)* | Add near doubles of ***two-digit numbers***),  *(e.g. 18 + 16, 38 + 37, 60 + 70, 50 + 70)*  *partition: double and adjust* | Add and subtract a ***three-digit number*** and ***ones***, including ***crossing tens boundary***, *(e.g. 472 + 4, 476 – 4, 389 + 5, 384 - 7)* |

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| Add and subtract a ***two-digit*** or ***three-digit*** ***multiple of 10***,  *(e.g. 120 – 40, 140 + 150, 370 – 180)*  *use knowledge of place value and related calculations, (e.g. work out 140 + 150 = 290 using 14 + 15 = 29)* | Add and subtract a ***three-digit number*** and ***tens***, including ***crossing the hundreds boundary***,  *(e.g. 356 + 30, 356 + 80)* | Add and subtract a ***three-digit number*** and ***hundreds***  *(e.g. 459 + 300)* | Add and subtract two ***three-digit multiples of 10,***  *(e.g. 620 – 380, 350+ 360)* | Add and subtract a ***near multiple of 10 or 100*** to any ***two-digit or three-digit number***,  *(e.g. 235 + 198)* | Find the difference between ***near multiples of 100***,  *(e.g. 607 – 588)* |

**Written Procedures**

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| Add numbers with ***two digits*** and ***three digits***, using formal written methods of columnar addition, ***without crossing boundaries*** | Add numbers with ***two digits*** and ***three digits***, using formal written methods of columnar addition , ***crossing boundaries*** | Subtract numbers with ***two digits*** and ***three digits***, using formal written methods of columnar subtraction, ***without crossing boundaries*** | Subtract numbers with ***two digits*** and ***three digits***, using formal written methods of columnar subtraction, ***crossing boundaries*** |

**Year 3 Progression Multiplication and Division**

**Multiplication and Division Big Ideas**

* It is important for children not just to be able to ***chant their multiplication tables*** but also to understand what the ***facts in them mean***, to be able to ***use these facts to figure out others*** and to use in ***problems***. It is also important for children to be able to ***link facts within the tables*** (e.g. 5× is half of 10×).
* They understand what ***multiplication means***, see ***division*** as both ***grouping*** and ***sharing***, and see ***division as the inverse of multiplication***.

**Multiplication & Division: rapid recall facts, mental skills, strategies & methods**

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| Recall and use multiplication and division facts for the ***3*** ***multiplication tables*** | Recall and use multiplication and division facts for the ***4*** ***multiplication tables*** | Recall and use multiplication and division facts for the ***8*** ***multiplication tables*** | Doubles of all numbers ***1 to 100*** and corresponding ***halves*** | Halve any ***multiple of 10*** up ***to 200*** *(e.g. halve 170)*  *partition: when halving, halve the tens and ones separately, then recombine* | Multiply ***one-digit*** and ***two-digit numbers*** ***by 10***, *(e.g. 3 x 10, 46 x 10)*  *recognise that when multiplying by 10 the digits move one place to the left and zero is used as a place holder* | Multiply ***one-digit*** and ***two-digit numbers*** ***by 100***,  *(e.g. 3 x 100, 46 x 100)*  *recognise that when multiplying by 100 the digits move two places to the left and zero is used as a place holder* |

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| Solve missing number problems using table facts from the ***2, 5, 10, 3, 4*** & ***8*** times tables | Solve positive integer scaling problems and corresponding problems in which n objects are connected to m objects | Write and calculate mathematical statements for multiplication and division facts from the ***2, 5, 10, 3, 4*** & ***8*** times tables | Identify the remainder when dividing by ***2, 5*** and ***10*** | Recognise that ***tenths*** arise from dividing an object into ***10*** equal parts and in dividing ***one-digit numbers*** or quantities ***by 10*** |

**Written Procedures**

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| Multiply a ***two-digit number*** by a ***one-digit number*** using an informal layout *(e.g. grid method)* | Multiply a ***two-digit number*** by a ***one-digit number*** using a formal layout *(e.g. short method)* |

**Year 4**

**Year 4 Progression Place Value and Counting**

**Place Value Big Ideas**

* Imagining the ***position*** of numbers on a ***horizontal number line*** helps us to ***order*** them: the number to the right on a number line is the larger number. So 5 is greater than 4, as 5 is to the right of 4. But –4 is greater than –5 as –4 is to the right of –5.
* ***Rounding*** numbers in context may mean ***rounding up or down***. Buying packets of ten cakes, we might round up to the nearest ten to make sure everyone gets a cake.
* ***Estimating*** the number of chairs in a room for a large number of people we might round down to estimate the number of chairs to make sure there are enough.
* We can think of ***place value*** in ***additive terms***: 456 is 400 + 50 + 6, or in ***multiplicative terms***: one hundred is ten times as large as ten.

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| Count backwards through ***zero*** to include ***negative numbers*** | Count in multiples of ***6*** | Count in multiples of ***7*** | Count in multiples of ***9*** | Count in multiples of ***25*** | Count in multiples of ***1000*** | Find ***1000*** more and less than a given number | Compare and order numbers ***beyond 1000*** | Compare numbers with the ***same number of decimals places (1.d.p.)*** | Compare numbers with the ***same number of decimal places (2.d.p.)*** |

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| Read Roman numerals ***to 100*** | Recognise the place value of each digit in a ***four-digit number***  *(e.g. thousands, hundreds, tens, ones)* | Recognise the place value of digits in numbers to ***one decimal place***  *(e.g. ones, tenths)* | Recognise the place value of digits in numbers to ***two decimal places***  *(e.g. ones, tenths, hundredths)* | Identify the value of each digit in numbers to ***one decimal place*** | Round any number to the ***nearest 10***, ***up to 10 000*** | Round any number to the ***nearest 100***, ***up to 10 000*** | Round any number to the ***nearest 1000***, ***up to 10 000*** | Round decimals with ***one decimal place*** to the ***nearest whole number*** |

**Year 4 Progression: Addition & Subtraction**

**Addition & Subtraction Big Ideas**

* It helps to ***round*** numbers before carrying out a calculation to get a ***sense of the size of the answer***. For example, 4786 – 2135 is close to 5000 – 2000, so the answer will be around 3000. Looking at the numbers in a calculation and their ***relationship*** to each other can help make calculating easier. For example, 3012 – 2996. Noticing that the numbers are close to each other might mean this is more easily calculated by thinking about subtraction as difference.

**Addition & Subtraction: derive & recall, mental skills, strategies & methods**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Sums and differences of pairs of ***multiples of 10, 100 &1000*** | What must be added to any ***three-digit number*** to make the next ***multiple of 100***, *(e.g. 521 + □ = 600)* | Add ***near doubles*** of ***three-digit numbers***,  *(e.g. 138 + 137)* | What must be added to any ***four-digit number*** to make the ***next multiple of 1000,***  *(e.g. 4087 + □ = 5000)* | Find the difference between ***near multiples of 1000****, (e.g. 6070 – 4087)* |

**Written Procedures**

|  |  |
| --- | --- |
| Add numbers with up to ***four digits***, using ***formal written methods*** of columnar addition | Subtract numbers with up to ***four digits***, using ***formal written methods*** of columnar subtraction |

**Year 4 Progression Multiplication and Division**

**Multiplication and Division Big Ideas**

* It is important for children not just to be able to ***chant their multiplication tables*** but to understand what the facts in them ***mean***, to be able to ***use these facts to figure out others*** and to use them in ***problems***.
* It is also important for children to be able to ***link facts within the tables*** (e.g. 5× is half of 10×).
* They understand what ***multiplication means*** and see ***division*** as both ***grouping*** and ***sharing***, and to see ***division as the inverse of multiplication.***
* The ***distributive law*** can be used to ***partition*** numbers in ***different ways*** to ***create equivalent calculations***. For example, 4 × 27 = 4 × (25 + 2) = (4 × 25) + (4 × 2) = 108.
* Looking for ***equivalent calculations*** can make calculating easier. For example, 98 × 5 is equivalent to 98 × 10 ÷ 2 or to (100 × 5) – (2 × 5). The array model can help show equivalences.

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| ***Year 4: Multiplication & Division: rapid recall facts, mental skills, strategies & methods***Recall and use multiplication and division facts for multiplication tables up to ***12 x 12*** | Square numbers to ***12 x 12*** | | Multiply by ***0*** | | Multiply and divide by ***1*** | | Factor pairs for table facts to ***12 x 12*** | | Halve any even number ***to 200*** | | Use place value, known and derived facts to multiply and divide mentally  *If we know 7x 8 = 56 can we use this to find 70 x 8, 7 x 80 etc.* | | Multiply together ***three numbers*** | | Use factor pairs and commutatively in mental calculations | | Solve problems involving multiplying and adding using the distributive law  *13 x 4 = (10 + 3) x 4 = (10 x 4) + (3 x 4) = 40 + 12 = 52* | |
| Solve problems involving multiplying and adding using integer scaling problems and harder correspondence problems such as n objects are connected to m objects | | Multiply a ***multiple of 10*** by a ***one-digit number*** *(e.g. 40 x 3)* | | Multiply numbers ***to 20*** by a ***one-digit number*** *(e.g. 17 x 3)* | | Divide a ***two-digit number*** by a ***one-digit number*** including dividing by ***1*** | | Multiply ***whole numbers*** by ***100*** | | Recognise that hundreds arise when dividing an object by one hundred and dividing tenths by ***10*** | | Find the effect of dividing a ***one-digit*** or ***two-digit number*** by ***10*** | | Find the effect of dividing a ***one-digit*** or ***two-digit number*** by ***100*** | | Multiply and divide ***two-digit numbers*** by ***4*** *(e.g. 26 x 4)* | | Multiply and divide ***two-digit numbers*** by ***8*** *(e.g. 96 ÷ 8)* |

**Written Procedures**

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| --- |
| Multiply a **three-digit number** by a **one-digit number** using a formal layout (e.g. short method) |

**Year 5**

**Year 5 Progression Place Value and Counting**

**Place Value Big Ideas**

* Large numbers of ***six digits*** are named in a ***pattern of three***: hundreds of thousands, tens of thousands, ones of thousands, mirroring hundreds, tens and ones.
* It is helpful to ***relate large numbers*** to ***real-world contexts***, for example the number of people that a local sports arena can hold.

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| Interpret negative numbers in context | Count forwards and backwards with positive and negative ***whole numbers***, including ***zero*** | | Count forwards and backwards in steps of ***powers of 10*** for any given number, ***up to 1 000 000*** | Compare and order numbers ***to at least***  ***1 000 000*** | Compare and order numbers up to ***three decimals places*** | | Read and recognise Roman numerals ***to 1000*** | Identify the value of each digit in numbers to ***two decimals places***  *(e.g. thousands, hundreds, tens, ones)* | Identify the value of each digit in numbers ***to at least***  ***1 000 000***  *(e.g. million, hundred thousand, ten thousand, thousand, hundred, ten, one)* | |  | | Round any number to the ***nearest 10***, ***up to***  ***1 000 000*** |
| Round any number to the ***nearest 100***, ***up to 1 000 000*** | | Round any number to the ***nearest 1000***, ***up to 1 000 000*** | | Round any number to the ***nearest 10 000***, ***up to 1 000 000*** | | Round any number to the ***nearest 1 00 00***, ***up to 1 000 000*** | | Round decimals with ***two decimal place*** to the nearest ***whole number*** | |  | | Round decimals with ***two decimal place*** to ***one decimal place*** | |

**Year 5 Progression: Addition & Subtraction**

**Addition & Subtraction Big Ideas**

* Before starting any calculation is it helpful to think about whether or not you are confident that you can ***do it mentally***. For example, 3689 + 4998 may be done mentally, but 3689 + 4756 may require ***paper and pencil.***
* Carrying out an ***equivalent calculation*** might be easier than carrying out the given calculation. For example 3682 – 2996 is equivalent to 3686 – 3000 (constant difference).

**Year 5: Addition & Subtraction: derive & recall, mental skills, strategies & methods**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| * What must be added to any ***four-digit number*** to make the next ***multiple of 1000*** | * Sums and differences of decimals to ***one decimal place***, *(e.g. 6.5 + 2.7, 7.8 – 1.3)* | Doubles and halves of decimals to ***one decimal place***,  *(e.g. half of 5.6, double 3.4)* | What must be added to a decimal to ***one decimal place*** to make the next **whole number**,  *(e.g. 7.2 + □ = 8)* | Factor pairs ***to 100*** |

**Written Procedures**

|  |  |
| --- | --- |
| Add numbers with ***more than four digits***, using ***formal written methods*** of columnar addition | Subtract numbers with ***up to four digits***, using ***formal written methods*** of columnar subtraction |

**Year 5 Progression Multiplication and Division**

**Multiplication and Division Big Ideas**

* Pupils have a firm understanding of what multiplication and division mean and have a range of strategies for dealing with large numbers, including both mental and standard written methods. They see the idea of factors, multiples and prime numbers as connected and not separate ideas to learn.
* They recognise how to use their skills of multiplying and dividing in new problem solving situations.
* Fractions and division are connected ideas: 36 ÷ 18 = 31/68 = 2; 18/36 = ½.
* Factors and multiples are connected ideas: 48 is a multiple of 6 and 6 is a factor of 48.

**Year 5: Multiplication & Division: rapid recall facts, mental skills, strategies & methods**

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Double ***three-digit multiples*** of ***10*** to ***500*** and find corresponding halves | Multiply and divide ***two-digit numbers*** by ***4*** and ***8*** | Multiply ***two-digit numbers*** by ***5*** and ***20*** | Multiply by ***25*** and ***50*** | Double ***three-digit multiples*** ***of*** ***10 to 500*** and find the corresponding halves | Multiply pairs of ***multiples of 10*** | Multiply ***whole numbers by 1000*** | Divide ***whole numbers by 1000*** | Multiply numbers involving ***decimals*** by ***10, 100 and 1000*** | Divide numbers involving ***decimals*** by ***10, 100 and 1000*** | Identify ***multiples*** and ***factors*** |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Find all ***factor pairs*** of a number | Find ***common factors*** of two numbers | Know the vocabulary of ***prime numbers***, ***prime factors*** and ***composite (non-prime) numbers***  use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers | Establish whether a number ***up to 100*** is ***prime*** | Recall ***prime*** numbers  ***to 19*** | Recognise ***square numbers*** | Use ***square numbers*** and the notation for squared (2) | Recognise ***cube numbers*** | Use ***cube numbers***, and the notation for cubed (3) |

**Written Procedures**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Multiply numbers up to ***4 digits*** by a ***one- digit number*** using a ***formal*** written method | Multiply numbers up to ***4 digits*** by a ***two-digit number*** using a ***formal*** written method of ***long multiplication*** | Divide numbers up to ***3 digits*** by a ***one-digit number*** using the ***formal*** written method of ***short division without remainders*** | Divide numbers up to ***3 digits*** by a ***one-digit number*** using the ***formal*** written method of ***short division*** and ***interpret remainders*** appropriately for the context | Divide numbers up to ***4 digits*** by a ***one-digit number*** using the ***formal*** written method of ***short division*** and ***interpret remainders*** appropriately for the context |

**Year 6**

**Year 6 Progression Place Value and Counting**

**Place Value Big Ideas**

* For ***whole numbers***, the ***more digits*** a number has, the ***larger it must be***: any 4-digit whole number is larger than any 3-digit whole number. But this is ***not true of decimal numbers***: having more digits does not make a decimal number necessarily bigger. For example, 0·5 is larger than 0·35.
* ***Ordering decimal numbers*** uses the same process as for whole numbers i.e. we look at the ***digits*** in matching places in the numbers, starting from the place with the ***highest value*** i.e. from the left. The number with the higher different digit is the higher number. For example, 256 is greater than 247 because 256 has 5 tens but 247 has only 4 tens. Similarly 1·0843 is smaller than 1·524 because 1·0843 has 0 tenths but 1·524 has 5 tenths.

**Year 6: Place Value and Counting**

|  |  |  |  |
| --- | --- | --- | --- |
| Order and compare numbers ***up to 10 000 000*** | Determine the value of digit in numbers ***up to***  ***10 000 000*** | Round any ***whole number*** to a required degree of accuracy | Use negative numbers in context, and calculate intervals across zero |

**Year 6 Progression: Addition & Subtraction**

**Addition & Subtraction Big Ideas**

* Deciding which calculation method to use is supported by being able to ***take apart*** and ***combine*** numbers in ***many ways***. For example, calculating 8·78 + 5·26 might involve calculating 8·75 + 5·25 and then adjusting the answer.
* The ***associative rule*** helps when adding three or more numbers: 367 + 275 + 525 is probably best thought of as 367 + (275 + 525) rather than (367 + 275) + 525.

**Year 6: Addition & Subtraction: derive & recall, mental skills, strategies & methods**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| * Addition and subtraction facts for ***multiples of 10 to 1000,*** * *(e.g. 650 + □ = 930)* | * Addition and subtraction facts for ***decimals numbers*** with ***one decimal place,*** * *(e.g. □ – 1.4 = 2.5)* | What must be added to a ***decimal*** with up to ***two decimal places*** to make the ***next whole number***,  *(e.g. 7.26 + □ = 8)* | Add or subtract pairs of ***decimals,*** to ***two decimal places***,  *(e.g. 0.7 + 3.38)*  *count on and back in ones, tenths, hundredths etc.* | Find doubles of decimals, to ***one decimal place***,  *(e.g. 1.6 + 1.6)* | Add near doubles of ***decimals,*** to ***one decimal place***, *(e.g. 2.5 + 2.6)* | Add or subtract decimals, to ***one decimal place*** that are nearly ***whole numbers***, *(e.g. 4.3 + 2.9, 6.5 – 3.8)* |

**Written Procedures**

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| --- | --- |
| Add numbers with ***more than four digits***, using ***formal written methods*** of columnar addition | Subtract numbers with ***up to four digits***, using ***formal written methods*** of columnar subtraction |

**Year 6 Progression Multiplication and Division**

**Multiplication and Division Big Ideas**

* Standard written algorithms use the conceptual structures of the mathematics to produce efficient methods of calculation.
* Standard written multiplication method involves a number of partial products. For example, 36 × 24 is made up of four partial products 30 × 20, 30 × 4, 6 × 20, 6 × 4.
* There are connections between factors, multiples and prime numbers and between fractions, division and ratios.

**Year 6: Multiplication & Division: rapid recall facts, mental skills, strategies & methods**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Double ***decimals*** to ***one decimal place*** and find corresponding halves | Divide by ***25*** and ***50*** | Multiply ***pairs of multiples of 10 and 100*** | Multiply ***one-digit numbers*** with ***up to two decimal places*** by ***whole numbers*** | Divide ***one-digit numbers*** with ***up to two decimal places*** by ***whole numbers*** | Perform mental calculations including with ***mixed operations*** and ***large numbers*** | Multiply numbers by ***10, 100 and 100*** where the answers are ***up to three decimal places*** | Divide numbers by ***10, 100 and 100*** where the answers are ***up to three decimal places*** | Identify common factors, common multiples and prime numbers |

**Written Procedures**

|  |  |  |  |
| --- | --- | --- | --- |
| Multiply ***multi-digit numbers*** ***up to four digits*** by a ***two-digit whole number*** using the ***formal*** written method of ***long multiplication*** | Divide numbers ***up to four digits*** by a ***two-digit whole number*** using the ***formal*** written method of ***short division*** and ***interpret remainders*** and whole number remainders, fractions or by rounding, as appropriate for the context | Where appropriate for the context divide numbers ***up to four digits*** by a ***two-digit whole number*** using the ***formal*** written method of ***long division*** and ***interpret remainders*** and whole number remainders, fractions or by rounding, as appropriate for the context | Use written division methods in cases where the answer has ***up to two decimal places*** |

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|  | **Year 1** | **Year 2** | **Year 3** | **Year 4** | **Year 5** | **Year 6** |
| **Addition** | Combine two parts together to make a whole ( part – part – whole)  Adding by counting on from the bigger number  Regroup a 10.  All the above to 20, 40 and then 100. | Adding 3 single digits  Column method no regrouping  By the end of the year regrouping in ones and tens. | Column Method  Regrouping  Up to 4 digits | Column Method  Regrouping  Up to 4/5 digits | Column Method  Regrouping  With more than 4 digits  Decimals- with the same amount of decimal places. | Column Method  Regrouping  With more than 4 digits  Decimals- with different amounts of decimal places. |
| **Subtraction** | Taking away ones  Counting back  Part whole model  Make 10  All the above to 20, 40 and then 100.  Regroup numbers into tens and ones | Part whole model  Column method no regrouping  By the end of the year regrouping in ones and tens. | Column Method  Regrouping  Up to 4 digits | Column Method  Regrouping  Up to 4/5 digits | Column Method  Regrouping  With more than 4 digits  Decimals- with the same amount of decimal places. | Column Method  Regrouping  With more than 4 digits  Decimals- with different amounts of decimal places. |
| **Multiplication** | Doubling  Counting in multiples  Arrays with support  Relate repeated addition to multiplication concept  Multiplication stories | Doubling  Counting in multiples  Arrays - number of groups by the number of items.  Repeated addition | Counting in multiples  Repeated addition  Begin column multiplication | Column multiplication  2 and 3 digit by 1 digit | Column Multiplication  Up to 4 digits  By 1 and 2 digits.  Short multiplication with decimals | Column Multiplication  By 2 digits  Decimals |
| **Division** | Sharing objects into groups  Division as grouping | Sharing objects into groups. | Division with arrays as a recap  Division with a remainder  Short division 2 digit by 1 digit – using concrete, pictorial and abstract. | Short division 3 digit by 1 digit – using concrete and pictorial.  Start work on interpreting remainders. | Short division (bus stop method)  Divide numbers up to 4 digits by 1 number equally and interpreting remainders. (As fractions and decimals)  Decimals | Short division (bus stop method)  Long division  Decimals  (C.P.A) |

|  |  |  |  |
| --- | --- | --- | --- |
| **Addition** | | | |
| Objective and Strategies | Concrete | Pictorial | Abstract |
| Combining two parts to make a whole: part- whole model | Use cubes to add two numbers together as a group or in a bar.    \\eha-fs-01\staff\Lisa.Lavelle\Documents\My Pictures\2018-01-10\IMGP0018.JPG  The part - part - whole model and using cubes to show this. Represent this in a variety of ways. | Represent this in a variety of ways | 4 + 3 = 7  10= 6 + 4  5  3  Use the part-part whole diagram as shown above to move into the abstract. |
| Starting at the bigger number and counting on | \\eha-fs-01\staff\Lisa.Lavelle\Documents\My Pictures\2018-01-10\003.JPG    Use numicon or bead strings to put the bigger number there already encourage that the children recognise that this is 10 and not counting it.  Count on using objects. | 12 + 5 = 17    Start at the larger number on the number line and count on in ones or in one jump to find the answer.  7 + 2 = 9 | 5 + 12 = 17  Place the larger number in your head and count on the smaller number to find your answer. |
| Regrouping to make 10 | 6 + 5 = 11    Start with the bigger number and use the smaller number to make 10.  Start with the bigger number and use the smaller number to make 10.      This can also be shown with Numicon resources. | Use pictures or a number line. Regroup or partition the smaller number to make 10.    Regroup or partition the smaller number to make 10.  \\eha-fs-01\staff\Lisa.Lavelle\Documents\My Pictures\2018-01-10\004.JPG | 6 + ? = 13  How many more do I need to make 13?  7 + 4= 11  If I am at seven, how many more do I need to make 10. How many more do I add on now? |
| Adding three single digits | 4 + 7 + 6= 17  Put 4 and 6 together to make 10. Add on 7.    Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.    E:\photos for calculation policy\Y2\101_1696.JPG | +  +  +            Add together three groups of objects. Draw a picture to recombine the groups to make 10. | Combine the two numbers that make 10 and then add on the remainder. |
| Column method- no regrouping | Use dienes to add the ones first and then the tens.  24 + 15=I:\Maths photos\IMG_2722.JPG  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0497.JPG    Move into hundreds  E:\photos for calculation policy\Y2\101_1684.JPG  Show the abstract setting out alongside this – model this as a class.  362 + 131 =  I:\Maths photos\IMG_2740.JPG  Move onto place value counters if appropriate.  E:\photos for calculation policy\Y2\101_1690.JPG  Show the abstract setting out alongside this – model this as a class. | After practically using the dienes and place value counters, children can draw the dienes or counters to help them to solve additions.  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0498.JPG      E:\photos for calculation policy\Y2\101_1689.JPG E:\photos for calculation policy\Y2\101_1693.JPG      Show the abstract setting out alongside this – model this as a class. | Children complete the calculations written using the column method.    I:\Maths photos\IMG_2739.JPG  I:\Maths photos\IMG_2741.JPG |
| Column method- regrouping | Make both numbers on a place value grid  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0500.JPG  Add up the ones and exchange 10 ones for one 10.    U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0501.JPG  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0502.JPG    U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0503.JPG  Regroup in the tens column  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0507.JPG  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0508.JPGI:\Maths photos\IMG_2744.JPG  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0510.JPG  Regroup in hundreds column  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0513.JPG U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0515.JPG U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0516.JPG  Move progressively to being able to regroup in all columns.  E:\photos for calculation policy\Y3\100_1232.JPG  E:\photos for calculation policy\Y3\100_1233.JPG  This can also be done with place value counters when children are ready.  As children move on to decimals, money and decimal place value counters can be used to support learning. | Children can draw a pictoral representation of the columns and dienes to further support their learning and understanding.    U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0504.JPG  U:\Maths updated - 2017\photos for calculation policy\117_0418\IMGP0511.JPG  I:\Maths photos\IMG_2732.JPG | I:\Maths photos\IMG_2743.JPG  I:\Maths photos\IMG_2745.JPG  As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here. |

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Subtraction** | | | | | | | | | | | |
|  | Taking away ones | | | | Use physical objects, counters, cubes etc. to show how objects can be taken away  I:\Maths photos\IMG_2746.JPG  I:\Maths photos\IMG_2748.JPGI:\Maths photos\IMG_2747.JPG | | | Cross out drawn objects to show what has been taken away | | 18 - 3= 15  8 – 2 = 6  Scaffold children to record their knowledge through number sentences with empty boxes. | |
| Counting back | | | | Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4    Use counters and move them away from the group as you take them away counting backwards as you go.http://3.bp.blogspot.com/-mFqQPE4k1TE/VGzRNnUu30I/AAAAAAAAAJM/12p6qvgkmoE/s1600/EvenOdd_ColoredCounters_Scattered.jpg | | | Count back on a number line or number track    Start at the bigger number and count back the smaller number showing the jumps on the number line.    This can progress all the way to counting back using two 2-digit numbers. | | Put 13 in your head, count back 4. What number are you at? **13 – 4 = □**  Abstract notation of number sentences. Include the use of missing boxes and the missing box in different places:  **26 – 13 = □**  **□ – 16 = 12**  **18 - □ = 7** | | |
| Find the difference | | | | Compare amounts and objects to find the difference.Image result for two towers of cubes    Use Numicon to show the difference. | | | Count on to find the difference. Draw bars to find the difference between 2 numbers.  http://image.slidesharecdn.com/intro-to-sm-1220840292402057-8/95/intro-to-singapore-math-13-728.jpg?cb=1345557040 | | Abstract notation of number sentences. Include the use of missing boxes and the missing box in different places:  26 – 13 = □  □ – 16 = 12  18 - □ = 7  Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches. | | |
| Part Part Whole Model | | | | Link to addition through the use of the ***PART-WHOLE*** model to help explain the inverse between addition and subtraction.    10 – 6 = □  If 10 is the ***WHOLE*** and 6 is one of the ***PARTS***. What is the other ***PART***? | | | Use a pictorial representation of objects to show the ***PART-PART-WHOLE*** model. | | Move to using numbers within the part whole model. | | |
| Make 10 | | | | 14 – 9 = □    Make 14 on the ten frame.  Take away the four first to make 10  Then takeaway one more so you have taken away 5.  You are left with the answer of 9. | | | Start at 13.  Take away 3 to reach 10.  Then take away the remaining 4 so you have taken away 7 altogether.  You have reached your answer. | | 16 – 8 = □  How many do we take off to reach the next 10?  How many do we have left to take off?  Abstract notation of number sentences. Include the use of missing boxes and the missing box in different places:  26 – 13 = □  □ – 16 = 12  18 - □ = 7  Missing box calculations will also provide an additional scaffold for pupils to begin their abstract recording. | | |
| Column method without regrouping | | | | IMGP0518IMGP0519  IMGP0520  Make the number then take away the number from the ones, tens and hundredsIMGP0519IMGP0519  IMGP0522IMGP0523  Follow the same process for hundreds, tens and ones and thousands, hundreds, tens and ones.  Show how you partition numbers to subtract. Again make the larger number first.  IMGP0523 | | | Draw the dienes or place value counters alongside the written calculation to help to show working.  First draw the biggest number. Cross out the ones and then cross out the tens. I:\Maths photos\IMG_2749.JPG    37 – 12 =  I:\Maths photos\IMG_2752.JPG326 – 112 = | | I:\Maths photos\IMG_2754.JPG  IMGP0525 | | |
| Column method with regrouping | | | | Use dienes to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.    Set out the larger number and take the number away          Once you have regrouped in the ones, tens, or hundreds then you can physically take away the number.  Make the larger number with the place value counters  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.    Now I can subtract my ones.  Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.    Now I can take away eight tens and complete my subtraction    Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount. | | | I:\Maths photos\IMG_2755.JPGDraw the dienes or place value counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.  342 – 128 = | | This can then be moved onto regrouping in all columns.  The children can then apply this method to much larger numbers and decimals. | | |
| **Multiplication** | | | | | | | | | | | |
| Doubling | | Use practical activities to show how to double and halve number.  Numicon  Dice    Domino | | | | Draw pictures to show how to double and halve a number. | | | | | Partition a number and then double each part before recombining it back together.    Link doubles to addition double number sentences.  Partition a number, and then halve each part before recombining it back together (as doubling example above). |
| Counting in multiples | | Count in multiples supported by concrete objects in equal groups. Skip counting.  Four groups of 5: 5, 10, 15, 20  ***4 x 5 = 20***  Four groups of 2: 2, 4, 6, 8    Actual fingers to support counting.    Numicon for counting in 5s and 2s and 10s.      Using coins: 1p, 2, 5p & 10p. | | | | Use a number line or pictures to continue support in counting in multiples. | | | | | Verbal count in multiples of a number aloud.  Write sequences with multiples of numbers (e.g. 2, 4, 6, 8, 10 or 5, 10, 15, 20, 25 , 30)  Include missing box sequences. |
| Repeated addition | | Use a range of different objects, real-life and linked to the wider curriculum and mathematical equipment, to add equal groups. | | | |  | | | | | Write number sentences to describe objects and/or pictorial representations.  2 + 2 + 2 + 2 + 2 = 10  Link repeated addition to multiplication:  2 + 2 + 2 = 6  2 x 3 = 6 |
| Arrays- showing commutative multiplication | | Create arrays using counters/ cubes to show multiplication sentences.http://www.australiancurriculumlessons.com.au/wp-content/uploads/2013/05/arrays-multiplication-division-lesson.jpg | | | | Draw arrays in different rotations to find commutative multiplication sentences.http://mathcentral.uregina.ca/QQ/database/QQ.02.06/maro1.1.gif | | | | | Use an array to write multiplication sentences and reinforce repeated addition.    5 + 5 + 5 = 15  3 + 3 + 3 + 3 + 3 = 15  5 x 3 = 15  3 x 5 = 15 |
| Grid method | | Show the link with arrays to first introduce the grid method.    4 rows of 10  4 rows of 3  Move on to using Base 10 to move towards a more compact method.    4 rows of 13    Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.      Fill each row with 126.    Add up each column, starting with the ones making any exchanges needed.      Then you have your answer. | | | | Children can represent the work they have done with place value counters in a way that they understand.  They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown belowhttp://www.highviewschool.org.uk/wp-content/uploads/2014/05/IMG_0499-300x225.jpg | | | | | Start with multiplying by one digit numbers and showing the clear addition alongside the grid.http://www.mumsnet.com/system/1/assets/files/000/006/988/6988/35010b289/original/328x164xgrid-method-explained-2.jpg.pagespeed.ic.zL-KyDdiL2.jpg  Moving forward, multiply by a 2 digit number showing the different rows within the grid method. |
| Column multiplication | | Children can continue to be supported by place value counters at the stage of multiplication.  64 x 3 =  https://primarysite-prod.s3.amazonaws.com/0c4eb252d34643748228179a3d582154_1x1.jpeg  column multiplication  column multiplication  column multiplication  column multiplication  It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.  column multiplication | | | | Children could then move onto drawing the counters.  I:\Maths photos\IMG_2757.JPG  IMGP0602  column multiplicationSKM_C554e18011016360_0002SKM_C554e18011016360_0002 | | | | | Start with long multiplication, reminding the children about lining up their numbers clearly in columns.      If it helps, children can write out what they are solving next to their answer.  http://ictedusrv.cumbria.ac.uk/maths/SecMaths/U1/images/pic018.gif  This moves to the more compact method.    SKM_C554e18011016360_0002SKM_C554e18011016360_0002 |
| **Division** | | | | | | | | | | | |
| Sharing objects into groups | | | I have 10 cubes, can you share them equally in 2 groups? | | | Children use pictures or shapes to share quantities.  6 teddies shared into 2 hoops.  6 ÷ 2 = 3 | | | | | Share 9 buns between three people.  9 ÷ 3 = 3 |
| Division as grouping | | | Divide quantities into equal groups.  Use cubes, counters, objects or place value counters to aid understanding or link to real-life equipment and those connected to the wider curriculum.        Place the ‘groups of 5’ on top or underneath the 20. How many ‘5s’? Make clear links to multiplication: 4 ‘5s’ - 5 x4. | | | If we have ten forks and we put them into groups of two, there are 5 groupshttp://gcamath3.weebly.com/uploads/9/1/4/0/9140392/200455_orig.jpg  Use a number line to show jumps in groups. The number of jumps equals the number of groups.    Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. | | | | | 10 ÷ 2 = 5  Include missing box calculations.  28 ÷ 7 = 4  Divide 28 into 7 groups. How many are in each group? |
| Division within arrays | | | Link division to multiplication by creating an array and thinking about the number sentences that can be created.  Eg 15 ÷ 3 = 5 5 x 3 = 15  15 ÷ 5 = 3 3 x 5 = 15 | | | Draw an array and use lines to split the array into groups to make multiplication and division sentences.  . | | | | | Find the inverse of multiplication and division sentences by creating four linking number sentences (fact families)  7 x 4 = 28  4 x 7 = 28  28 ÷ 7 = 4  28 ÷ 4 = 7 |
| Division with a remainder | | | 14 ÷ 3 =  Divide objects between groups and see how much is left over    20 ÷ 3 = 6 r 2  Bead string to model reminders. | | | Draw dots and group them to divide an amount and clearly show a remainder.    Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. | | | | | Complete written divisions and show the remainder using r.  20 ÷ 3 = 6 r 2  You may want to note down that 6 x 3 = 18 so 6 groups with 2 remainder  Include missing box calculations.http://amsi.org.au/teacher_modules/G7/G7_qt2%202.png |
| Short Division | | | Use place value counters to divide using the bus stop method alongside  42 ÷ 3=  Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.    We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14. | | | Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.http://www.studyzone.org/testprep/math4/d/division2.gif  Encourage them to move towards counting in multiples to divide more efficiently. | | | | | Begin with divisions that divide equally with no remainder.    Move onto divisions with a remainder.    Show remainders as decimals and fractions.    **Long division**  Partition the divisior to find multiples  To calculate long division the rule is: divide, multiply, subtract and drop down    Some children may use chunking as a way to calculate long division |